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ITERATIVE METHOD FOR THE ESTIMATION OF SHOT AND SEA FLOOR DEPTHS USING HYDROPHONE STREAMER DATA

Richard K. Brienzo



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Iterative method for the estimation of shot and sea floor depths using hydrophone streamer data

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ABSTRACT

When explosive charges are used as sound sources in marine refraction experiments, a streamer hydrophone is often trailed near the surface by the vessel which launches the charges. The output of the streamer hydrophone is used to monitor the detonations, and after correcting for the time spent in traveling from the shot to the streamer hydrophone, provides a time origin for each shot. If three arrivals are present (the direct water path and two reflections), then information about the shot depth and sea floor depth may also be obtained.

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Since the ship that launches the charges generally steams away from the drop site, incorrect results are obtained if vertical raypaths are assumed. In this report, a method which does not assume vertical raypaths is derived to find shot and sea floor depths. Inputs are the distance traveled from the drop site when the detonation first appears on the streamer hydrophone output, and the time differences between the three arrivals. A Fortran-77 program which implements the method is listed and its use is illustrated on data obtained during a refraction experiment conducted on the Monterey Deep-Sea Fan.

Iterative method for the estimation of shot and sea floor depths using hydrophone streamer data

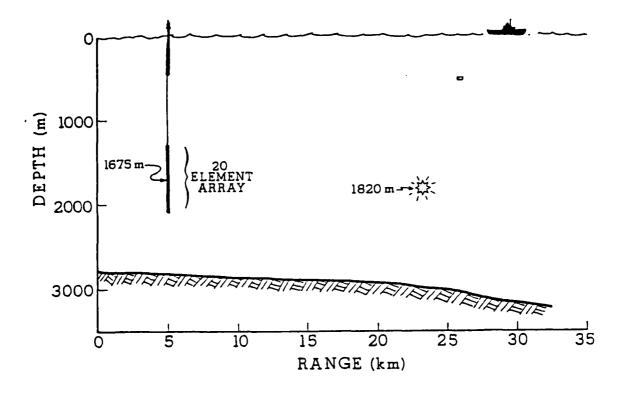
1. Introduction

Marine refraction experiments are conducted using an impulsive airgun or explosive charges as a sound source. A typical experiment (Figure 1) might have one vessel utilized as a receiving station with a hydrophone, or a hydrophone array, deployed in the water column. A second vessel (called the shooting ship) steams away from the receiving ship and drops explosive charges into the water. Acoustic energy travels along numerous paths in going from the shot to the array. Since there is a general increase in sound speed with depth in the sediment, much of the energy that enters the sediment is refracted back into the water column and received at the array. Analysis of this received data yields information about the velocity and attenuation structure in the sediment.

When explosive charges are used, a hydrophone (called a streamer hydrophone) is often trailed near the surface by the shooting ship while charges are being launched. Streamer hydrophones are used during marine refraction experiments to monitor the detonations. Output from the streamer hydrophone is telemetered to the receiving ship via a radio voice channel and recorded along with data from the array. Streamer hydrophone output indicates when the shot detonates, and after correcting for the time spent in traveling from the shot to the streamer hydrophone, provides a time origin for each shot. Additionally, if three arrivals can be seen on the streamer hydrophone output, estimates of the shot and sea floor depths may be obtained.

An example of this type of data is given in Figure 2. The top nine time series are the array data and the bottom time series is the output of the streamer hydrophone. In the streamer data, the pulse at 0.32 seconds is the arrival which travels from the shot directly to the streamer hydrophone. The second arrival (at 1.51 seconds) is a bottom reflection, and the third arrival (at 3.69 seconds) is a surface reflection - bottom reflection. Energy appears at the array approximately 14.7 seconds after it arrives at the streamer hydrophone.

Ray paths corresponding to the arrivals on the streamer channel are illustrated in Figure 3(a). The first arrival is denoted (D), the second arrival (B), and the third (SB). The shooting ship moves a horizontal distance x from the drop site. The water column is modeled as a two layer medium. Layer boundaries are specified by the surface, shot depth (d_1) , and sea floor depth $(d_1 + d_2)$. Each layer is assumed to have a constant sound speed $(v_1 \text{ and } v_2)$. This is not a very good model of a velocity profile for the water column, but if mean values for each layer are found from the velocity profile, the method gives satisfactory results.



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Figure 1. Refraction experiment.

A hydrophone array is deployed at mid-depth in the water column. The shooting ship steams away from the array and drops explosive charges into the water. There is a general increase in sound speed with depth in the sediment, so much of the acoustic energy that enters the sediment is refracted back into the water column. Analysis of this data yields information about velocity and attenuation in the sediment.

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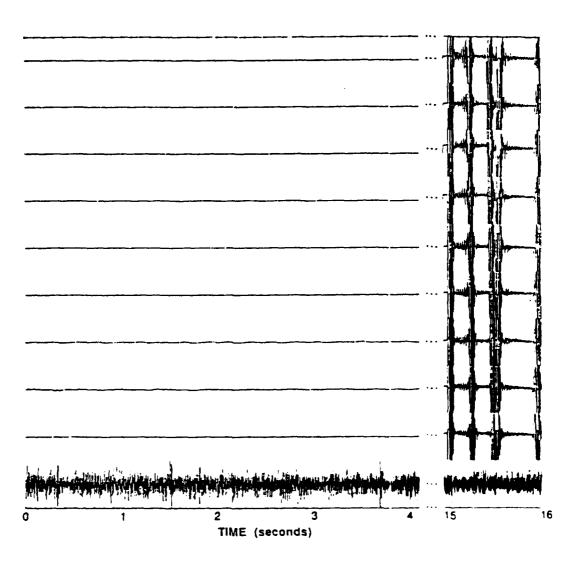


Figure 2. Hydrophone array and streamer data.

The top 9 time series are array data, bottom time series is the output of the streamer hydrophone.

Three arrivals are present on the streamer channel (at .32, 1.51, and 3.69 seconds). Energy appears at the array approximately 14.7 seconds after the first arrival at the streamer hydrophone.

2. Derivation

If the angles in Figure 3(a) are small (i.e. the ship does not travel very far from the drop site), then the raypaths are almost vertical, and the time difference between the second and third arrivals ($\Delta t_{2,3}$) is due to the extra distance that the (SB) ray travels in layer 1. In this case the third (SB) arrival reaches the hydrophone $2t_0$ seconds after the second (B) arrival (see Figure 3(b)). From this, the depth of the first layer is given by

$$d_1 = \frac{1}{2} \Delta t_{2,3} v_1$$

where $\Delta t_{2,3}$ is the time difference between the second (B) and third (SB) arrivals, and v_1 is the velocity in layer 1.

The depth of the second layer is found from the time differences between the direct (D) and bottom reflected (B) paths. If the angles are small, then t_1 and $(t_2 - t_3)$ are approximately the same, and it takes the bottom reflected path $2t_3$ seconds longer to reach the hydrophone. The depth of the second layer is then given by

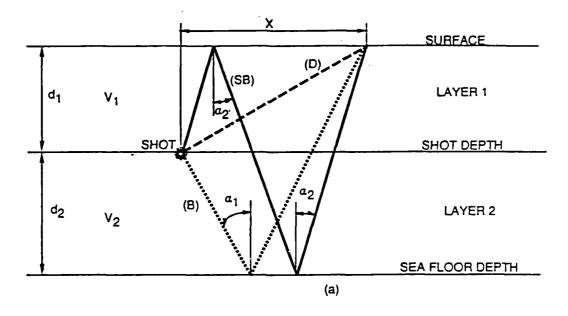
$$d_2 = \frac{1}{2} \Delta t_{1,2} v_2$$

where $\Delta t_{1,2}$ is the time different between the direct (D) and second (B) arrivals, and v_2 is the velocity in layer 2.

In general, the horizontal distance x will not be zero (it is not in the best interest of the ship's crew to remain over the site where an explosive charge has been dropped). This method will give incorrect depths for nonzero distances x since the raypaths are not vertical.

Non-zero ranges

When the distance x is not zero, it is necessary to correct for the difference between the vertical distances d_1 and d_2 and the actual length of the raypaths. Since the depths d_1 and d_2 are unknown, the angles α_1 and α_2 are also unknown. The strategy will be to initially use the values \hat{d}_1 and \hat{d}_2 found by assuming vertical ray paths. Then, given x, corresponding angles α_1 and α_2 may be found. These angles are then used to include the effect of non-vertical ray paths in the calculation of depths d_1 and d_2 . Since \hat{d}_1 and \hat{d}_2 are not exact, α_1 and α_2 are not exact, and the calculated d_1 and d_2 will not be correct, but will be closer to the correct values than \hat{d}_1 and \hat{d}_2 . This procedure may be iterated using the calculated values d_1 and d_2 as the new estimates of \hat{d}_1 and \hat{d}_2 to obtain successively closer values to the correct depths.



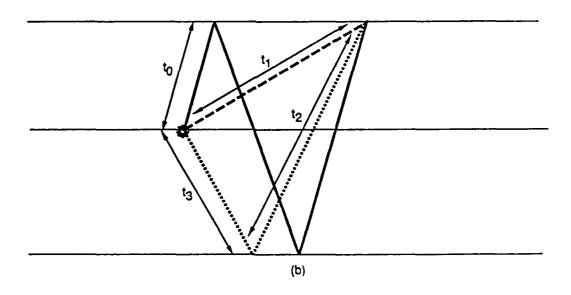


Figure 3.

Relations for finding α_1 and α_2 from d_1 and d_2 are derived from the geometry of the ray paths. From Figure 4(a),

$$\tan(\alpha_1) = \frac{x}{d_1 + 2d_2} \qquad \alpha_1 = \arctan\left(\frac{x}{d_1 + 2d_2}\right)$$
 (1)

From Figure 4(b),

$$\tan(\alpha_2) = \frac{x_1}{d_1}$$
 and $\tan(\alpha_2) = \frac{1/2x_2}{(d_1 + d_2)}$ (2)

Equating these last two expressions and solving for x_2 gives

$$\frac{x_1}{d_1} = \frac{1/2x_2}{(d_1 + d_2)}$$

$$x_2 = \frac{2(d_1 + d_2)x_1}{d_1}$$

The total distance is $x = x_1 + x_2$. Using this to find x_1 gives

$$x_1 = x - x_2$$

$$x_1 = x - \frac{2(d_1 + d_2)x_1}{d_1}$$

$$x = x_1 \left[1 + \frac{2(d_1 + d_2)}{d_1} \right]$$

$$x_1 = \frac{x}{\left[1 + \frac{2(d_1 + d_2)}{d_1}\right]}$$

From this and equation (2),

$$\tan(\alpha_2) = \frac{x_1}{d_1} = \frac{x}{d_1 \left[1 + \frac{2(d_1 + d_2)}{d_1}\right]} = \frac{x}{(3d_1 + 2d_2)}$$

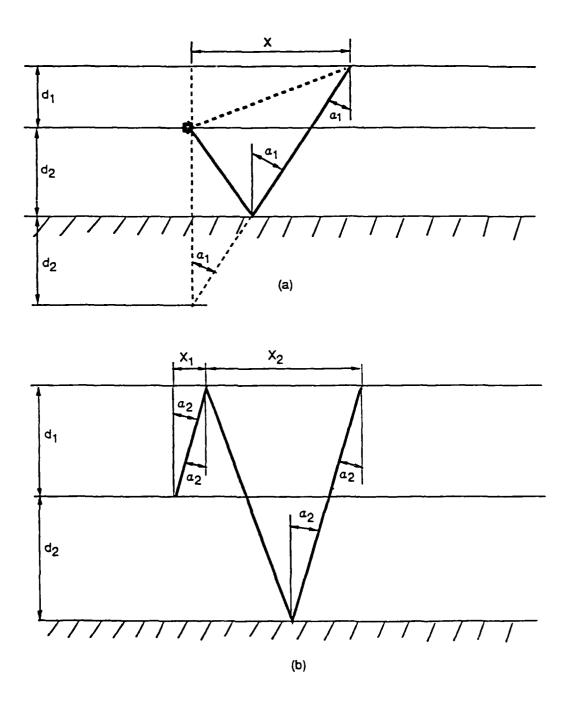


Figure 4.

Therefore,

$$\alpha_2 = \arctan\left(\frac{x}{3d_1 + 2d_2}\right) \tag{3}$$

These equations (summarized below) give the angles α_1 and α_2 in terms of the depths d_1 and d_2

$$\alpha_1 = \arctan\left(\frac{x}{d_1 + 2d_2}\right) \tag{4a}$$

$$\alpha_2 = \arctan\left(\frac{x}{3d_1 + 2d_2}\right) \tag{4b}$$

Estimated depths (assuming vertical raypaths) \hat{d}_1 and \hat{d}_2 may be substituted into (4) to obtain estimates of the angles α_1 and α_2 . These angles α_2 then be used along with the time delays to find depths d_1 and d_2 . Since these depths are now calculated using non-vertical ray paths, they will be closer to the correct shot and seafloor depths. Expressions for d_1 and d_2 in terms of α_1 , α_2 , and the time delays are derived next.

From Figure 5, the time difference between the third and second arrivals is

$$= \left(\frac{2d_1}{v_1\cos(\alpha_2)} + \frac{2d_2}{v_2\cos(\alpha_2)} + \frac{d_1}{v_1\cos(\alpha_2)}\right) - \left(\frac{2d_2}{v_2\cos(\alpha_1)} + \frac{d_1}{v_1\cos(\alpha_1)}\right)$$

$$= \left(\frac{3}{\cos(\alpha_2)} - \frac{1}{\cos(\alpha_1)}\right) \frac{d_1}{v_1} + \left(\frac{2}{\cos(\alpha_2)} - \frac{2}{\cos(\alpha_1)}\right) \frac{d_2}{v_2}$$

 α_1 and α_2 are known (from equations (4a) and (4b)), so define the constants

$$k_1 = \frac{3}{\cos(\alpha_2)} - \frac{1}{\cos(\alpha_1)}$$

 $\Delta t_{23} = (2t_0 + 2t_3 + t_4) - (2t_1 + t_2)$

$$k_2 = \frac{2}{\cos(\alpha_2)} - \frac{2}{\cos(\alpha_1)}$$

then $\Delta t_{2,3}$ may be written as

$$\Delta t_{2,3} = \frac{k_1}{v_1} d_1 + \frac{k_2}{v_2} d_2 \tag{5}$$

The difference between the second and first arrivals is (Figure 5)

$$\Delta t_{1,2} = (2t_1 + t_2) - t_D$$

$$= \left(\frac{2d_2}{v_2 \cos(\alpha_1)} + \frac{d_1}{v_1 \cos(\alpha_1)}\right) - \frac{1}{v_1} \sqrt{d_1^2 + x^2}$$

$$= \left(\frac{2d_2}{v_2} + \frac{d_1}{v_1}\right) \frac{1}{\cos(\alpha_1)} - \frac{1}{v_1} \sqrt{d_1^2 + x^2}$$
 (6)

Solving (5) for d_1 gives

$$d_1 = \frac{v_1}{k_1} \Delta t_{2,3} - \frac{k_2}{k_1} \frac{v_1}{v_2} d_2 \tag{7}$$

The expression in (7) may be used in (6) for the d_1 enclosed in parentheses (d_1 under the radical will be replaced later)

$$\Delta t_{1,2} = \frac{1}{\cos(\alpha_1)} \left[\frac{2d_2}{v_2} + \frac{1}{v_1} \left(\frac{v_1}{k_1} \Delta t_{2,3} - \frac{k_2}{k_1} \frac{v_1}{v_2} d_2 \right) \right] - \frac{1}{v_1} \sqrt{d_1^2 + x^2}$$

$$= \frac{1}{\cos(\alpha_1)} \left[\frac{2d_2}{v_2} + \frac{\Delta t_{2,3}}{k_1} - \frac{k_2}{k_1} \frac{d_2}{v_2} \right] - \frac{1}{v_1} \sqrt{d_1^2 + x^2}$$

$$= d_2 \left(\frac{2}{v_2 \cos(\alpha_1)} - \frac{k_2}{k_1 v_2 \cos(\alpha_1)} \right) + \frac{\Delta t_{2,3}}{k_1 \cos(\alpha_1)} - \frac{1}{v_1} \sqrt{d_1^2 + x^2}$$

$$\sqrt{d_1^2 + x^2} = \left(\frac{2v_1}{v_2 \cos(\alpha_1)} - \frac{k_2 v_1}{k_1 v_2 \cos(\alpha_1)}\right) d_2 + \left(\frac{v_1 \Delta t_{2,3}}{k_1 \cos(\alpha_1)} - v_1 \Delta t_{1,2}\right)$$
(8)

To simplify the above expression, define

$$c_1 = \frac{2v_1}{v_2\cos(\alpha_1)} - \frac{k_2v_1}{k_1v_2\cos(\alpha_1)}$$

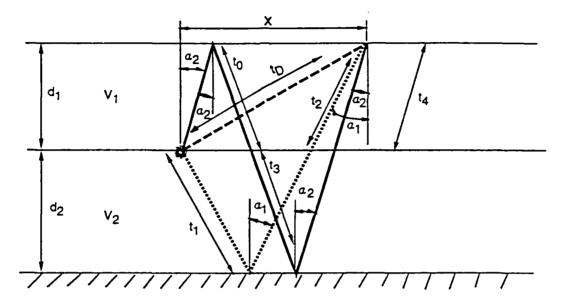


Figure 5.

$$c_2 = \frac{v_1 \Delta t_{2,3}}{k_1 \cos(\alpha_1)} - v_1 \Delta t_{...}$$

and (8) may be written as

$$\sqrt{d_1^2 + x^2} = c_1 d_2 + c_2 \tag{9}$$

Next, substitute the expression for d_1 from (7) into (9) and square both sides.

$$\left(\frac{v_1}{k_1}\Delta t_{2,3} - \frac{k_2}{k_1}\frac{v_1}{v_2}d_2\right)^2 + x^2 = c_1^2 d_2^2 + 2c_1c_2d_2 + c_2^2$$

$$\left(\frac{v_1 \Delta t_{2,3}}{k_1}\right)^2 - \frac{2v_1^2 k_2 \Delta t_{2,3}}{k_1^2 v_2} d_2 + \left(\frac{k_2 v_1}{k_1 v_2}\right)^2 d_2^2 + x^2 = c_1^2 d_2^2 + 2c_1 c_2 d_2 + c_2^2$$

$$\left[\left(\frac{k_2 v_1}{k_1 v_2} \right)^2 - c_1^2 \right] d_2^2 + \left[-\frac{2 v_1^2 k_2 \Delta t_{2,3}}{k_1^2 v_2} - 2 c_1 c_2 \right] d_2 + \left[\left(\frac{v_1 \Delta t_{2,3}}{k_1} \right)^2 - c_2^2 + x^2 \right] = 0$$

This is in the form $a d_2^2 + b d_2 + c = 0$ and the quadratic formula may be used to solve for d_2 . Once d_2 is known, d_1 may be found from equation (7)

$$d_1 = \frac{v_1}{k_1} \Delta t_{2,3} - \frac{k_2}{k_1} \frac{v_1}{v_2} d_2$$

This estimate of the shot depth (d_1) and seafloor depth (d_1+d_2) includes the effect of non-vertical ray paths since angles α_1 and α_2 were used in the calculation. These values of d_1 and d_2 are then used to calculate new values for α_1 and α_2 , and the process is repeated. d_1 and d_2 should converge to constants within a few iterations.

3. Algorithm

Required inputs

- a) differences in time of arrivals $\Delta t_{1,2}$ and $\Delta t_{2,3}$.
- b) distance the ship has traveled from the drop site at the time of detonation.

$$x = (\text{ship's speed}) \times (\text{detonation time } - \text{ time charge was launched}).$$

c) average sound speed v_1 and v_2 in layers 1 and 2.

Outputs

- a) shot depth (d_1)
- b) distance from shot to sea floor (d_2) . Bottom depth is then given by $d_1 + d_2$.
- 1. Find approximate values for d_1 and d_2 by assuming vertical raypaths

$$\hat{d}_1 = \frac{1}{2} \Delta t_{2,3} v_1$$

$$\hat{d}_2 = \frac{1}{2} \Delta t_{1,2} v_2$$

2. Calculate α_1 and α_2

$$\alpha_1 = \arctan(\frac{x}{\hat{d_1} + 2\hat{d_2}})$$

$$\alpha_2 = \arctan(\frac{x}{3\,\hat{d}_1 + 2\,\hat{d}_2})$$

3. Calculate constants k_1 and k_2

$$k_1 = \frac{3}{\cos(\alpha_2)} - \frac{1}{\cos(\alpha_1)}$$

$$k_2 = \frac{2}{\cos(\alpha_2)} - \frac{2}{\cos(\alpha_1)}$$

4. Calculate constants c_1 and c_2

$$c_1 = \frac{2v_1}{v_2\cos(\alpha_1)} - \frac{k_2v_1}{k_1v_2\cos(\alpha_1)}$$

$$c_2 = \frac{v_1 \Delta t_{2,3}}{k_1 \cos(\alpha_1)} - v_1 \Delta t_{1,2}$$

5. Calculate coefficients for the quadratic formula

$$a = \left(\frac{k_2 v_1}{k_1 v_2}\right)^2 - c_1^2$$

$$b = -\frac{2v_1^2 k_2 \Delta t_{2,3}}{k_1^2 v_2} - 2c_1 c_2$$

$$c = \left(\frac{v_1 \Delta t_{2,3}}{k_1}\right)^2 - c_2^2 + x^2$$

6. Find depths d_2 and d_1

$$d_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$d_1 = \frac{v_1}{k_1} \Delta t_{2,3} - \frac{k_2}{k_1} \frac{v_1}{v_2} d_2$$

7. Use d_1 and d_2 found in step 6 as the next \hat{d}_1 and \hat{d}_2 in step 2 and repeat.

After d_1 and d_2 have converged to constants, d_1 gives the shot depth and $(d_1 + d_2)$ gives the sea floor depth.

4. Example

A Fortran-77 program which implements the algorithm on page 12 is given in the Appendix. To illustrate its use, shot and seafloor depths will be found using the data presented in Figure 2. This particular data was obtained from a refraction experiment conducted on the Monterey Deep-Sea Fan in 1981 by the Marine Physical Laboratory. The research vessel FLIP (FLoating Instrument Platform) was moored in 2800 meters of water and utilized as a receiving station. A 20 element hydrophone array was deployed at mid-depth in the water column. The output of nine of these hydrophones along with the hydrophone streamer data is shown in Figure 2.

A Navy tug steamed away from FLIP and dropped explosive charges (Mk 94 SUS + 256# TNT) into the water at ranges between 3.5 km to 37 km from FLIP. As each shot was launched the radar range between FLIP and the tug, and the time, were recorded. Velocity of the tug was obtained from this information. The distance traveled from the drop site by the tug when the detonation arrived at the streamer hydrophone could then be calculated.

Mean velocities were obtained by integrating the velocity profile between the surface and 1820 meters for layer 1, and from 1820 meters to 2800 meters for layer 2.

The following input to the program

Enter mean velocity in layer 1 (meters/sec)

1485

Enter mean velocity in layer 2 (meters/sec)

Enter first and second arrival time difference (sec)

1.19

Enter second and third arrival time difference (sec)

2.18

Enter distance ship traveled from drop site (meters)

Enter maximum number of iterations to perform

10

produces the output

shot depth:

1788.4

sea floor depth: 3016.6

These are reasonable values for this experiment. The shot depth of 1788 meters is within the rather large uncertainty of the detonation device. A bathymetric map of the area shows the sea floor depth to be between 2800 meters at FLIP and 3200 meters at a range of 32 km from FLIP.

Acknowledgements

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Appendix: Fortran-77 program
Implements the algorithm on page 12
      Iterative method to find shot depth (d1) and seafloor depth (d1 + d2)
     from hydrophone streamer data.
С
    real k1, k2
   ---- Input -----
    write(6,*) 'Enter mean velocity in layer 1 (meters/sec)'
    read(5,*) v1
    write(6,*) 'Enter mean velocity in layer 2 (meters/sec)'
    read(5,*) v2
    write(6,*) 'Enter first and second arrival time difference (sec)'
   read(5,*) dt12
    write(6,*) 'Enter second and third arrival time difference (sec)'
    read(5,*) dt23
    write(6,*) 'Enter distance ship traveled from drop site (meters)'
    read(5,*) x
    write(6,*) 'Enter maximum number of iterations to perform'
    read(5,*) iter
                     Step 1.
   Estimate d1 and d2 by assuming ray paths are vertical
    dhat1 = .5 * dt23 * v1
    dhat2 = .5 * dt12 * v2
   ---- Refine the solution -----
    do 100 i=1,iter
                     Step 2.
     alpha1 = atan(x/(dhat1 + 2.*dhat2))
     alpha2 = atan(x/(3.*dhat1 + 2.*dhat2))
c
                     Step 3.
     k1 = 3./\cos(alpha2) - 1./\cos(alpha1)
```

 $k2 = 2./\cos(alpha2) - 2./\cos(alpha1)$

Step 4.

c2 = v1*dt23/(k1*cos(alpha1)) - v1*dt12

c1 = 2.*v1/(v2*cos(alpha1)) - k2*v1/(k1*v2*cos(alpha1))

```
С
                     Step 5.
     a = (k2*v1/(k1*v2))*(k2*v1/(k1*v2)) - c1*c1
     b = -2.*v1*v1*k2*dt23/(k1*k1*v2) - 2.*c1*c2
     c = x*x - c2*c2 + (v1*dt23/k1)*(v1*dt23/k1)
С
                     Step 6.
С
       Note: The solution to the quadratic formula is -b +/- sqrt ...
           For the data used in this example the minus sign gives
С
           the correct result.
     d2 = (-b - sqn(b*b-4.*a*c))/(2.*a)
     d1 = v1*dt23/k1 - k2*v1*d2/(k1*v2)
       Make these values of d1 and d2 the new estimates, and repeat.
     dhat1 = d1
     dhat2 = d2
100 continue
    ---- Output ----
     write(6,*) 'snot depth: ', d1
     write(6,*) 'sea floor depth: ', d1+d2
     stop
     end
```



